Some examples comparing static and dynamic network approaches in water resources allocation models for the rivers of high instability of flows

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Abstract

In this paper the authors compare two mathematical approaches to the problem of determination of optimal water resources allocation. We compare standard static approach based on static network flow model in pure or generalized network with the dynamic approach based on MDGNFM model presented in [WOJAS 2008]. This comparison is done in the framework of three worked examples of water system. We discuss the following aspects: a possibility to guarantee in the model the availability of the water which is allocated to user in analysed time period; the influence of a choice of the length of time step on the final result, a possibility to consider different summary times in water allocation paths. Comparative analysis can recommend the dynamic approach as more appropriate in the case of water systems of high instability of water flows.

Key words: dynamic networks, network flows, network optimisation, water resources allocation, water system balance

INTRODUCTION AND AIM OF THE STUDY

General methodology of surface water resources management is presented in [CBSiPBW Hydroprojekt 1992]. For quantitative allocation of surface water resources the simulation-optimisation models are used which consist in numerical mapping of areal structure of water system (river network) including the interaction between surface water resources and their use and protection as well as connection with underground water resources. For real–world problems of water resources management the static multi-period balance models were used [AHUJA et al. 1999; CHUNG et al. 1989; DAL LABADIE 2001; KINDLER 1975; SUN et al. 1995;]. This kind of approach – static network approach, does not take into account the real transit times of water transported through the water system and changes of values of inflows into the system in the analysed time period. Such assumptions (neglecting transit times and changes of inflow into the system) are acceptable for lowland rivers with relatively low dynamics of flow and time of water flow through the system much shorter than the fixed time period.

An alternative approach to modelling water balance problems – the dynamic network approach was proposed in paper by WOJAS [2008]. In the dynamic network model of water system presented in [WOJAS 2008], real transit times in water transhipment through segments of water system and the structure of changes of inflow into the system were considered. In
this paper we compare these two approaches using work examples of water systems of high instability and dynamics of river flow.

DIFFERENTIATION OF INSTABILITY RANGE AND RIVER FLOW DYNAMICS IN POLISH CONDITIONS

In Polish conditions there is a high differentiation of instability range and river flow dynamics. This is connected with the characteristics of outflow factors such as relief, soil type, area development – the degree of surface tightness in reception basin and meteorological conditions. Mountain and sub-mountain reception basins are characterised by a high instability range because of steep slopes and relatively small surface permeability and retention. Those factors are conducive to fast rain water runoff. A similar effect is observed in highly urbanised reception basins with large impermeable or slightly permeable areas from which water quickly flows down and is discharged through sewerage systems. The ratio of the lowest of annual low flows (LLQ) to the highest of annual high flows (HHQ) in a long run is one of the indicators used to describe the instability of flow processes in a given river section.

According to the practice of preparing water economy balances, simulation analyses are most frequently performed with a time step of one decade (meaning 10 years) and using averaged flow values of that period. The process of flow averaging may result in an incorrect estimation of the amount of disposable water resources, especially of mountain rivers where short-lasting and extremely high flow values (peaks) are observed. The process of averaging results is a significant overestimation of water resources amount in rainless periods when the actual flows in rivers are extremely low. This produces an effect similar to a retention basin that is not present in the existing system. The higher the dynamics of river flow process, the larger is the degree to which water resources values are misrepresented.

STATIC APPROACH IN THE NETWORK MODELLING OF WATER RESOURCES ALLOCATION

In the network models of surface water systems the spatial structure of a system is represented by a directed graph \((N, A)\) where \(N\) is a set of nodes and \(A\) is a set of ordered pairs of nodes called arcs.

An arc \((v, w)\) where \(v, w \in N\) is called outgoing from node \(v\) and entering node \(w\).

A directed path in a directed graph \((N, A)\) from node \(v_1\) to node \(v_n\) we call a sequence of nodes and arcs: \(v_1 - k_1 - v_2 - k_2 - \cdots - v_{n-1} - k_{n-1} - v_n\) where \(v_i \in N\) for \(i = 1, 2, \ldots, n\); and \(k_i = (v_i, v_{i+1}) \in A\) for \(i = 1, 2, \ldots, n-1\).

For each arc \(k\) we define two real nonnegative numbers: \(l(k)\) representing lower bound of capacity of arc and \(u(k)\) representing upper bound of capacity of arc.

We define additionally two nodes in a directed graph: \(s\) called source and \(t\) called sink. Water flow in a system is represented by real-valued function \(f(k)\) such that: \(l(k) \leq f(k) \leq u(k)\) for all \(k \in A\) and a sum of numbers \(f(k)\) for all outgoing arcs from a fixed node minus a sum of numbers \(f(k)\) for all arcs entering this node is non-negative for source, non-positive for sink and zero for others nodes.

Consequently, the absolute value of the difference of these sums is the same for source and for sink. The function \(f(k)\) is called static flow in the network from the source \(s\) to the sink \(t\). If water losses occur on the segments of the water system, modelling this kind of problem requires additional function \(g(k)\) denoted on the set \(A\) representing linear water losses in the system. A network with function \(g(k)\) we call generalised network in contrast to pure network (without function \(g(k)\)). Each water user in system is represented by node or arc or several arcs. If user is represented by a single arc then its upper bound of capacity represents user's water demand per time unit. Water resources allocation between water users is done in accordance to the established water use priorities. In accordance to the [CBSIPBW Hydroprojekt 1992] preserving priority means that the supply of water for a user located lower in the hierarchy cannot cause or increase water deficit in the more important user, i.e. that being higher in the hierarchy. To represent water use priorities we introduce real nonpositive function \(c(k)\) defined on the set \(A\) called cost function. Costs \(c(k)\) on the user's arcs should be defined to preserve hierarchy of water use priorities. Optimal water resources allocation problem can be formulated as an optimisation problem to determine a minimum cost static flow in pure or generalised network i.e. to determine static flow \(f(k)\) which minimises sum \(\sum c(k) f(k)\).

Popular optimisation algorithm to determine minimum cost static flow in pure network is „out-of-kilter” [FORD, FULKERSON 1969]. This algorithm has been successfully used to optimise the water resources allocation problems [BRENDECKE et al. 1989; CHUNG et al. 1989; KINDLER 1975; SABAT, CREEL 1991a, b]. For water resources allocation problems modelled by generalised network, EMNET algorithm was used. EMNET is based on the network simplex method. Also, CPLEX package was used [WOJAS 2008]. It solves the optimisation problems in LP form. In multiperiod models, after choosing the length of time step, the optimisation problem to determine minimum cost static flow is solved independently for each time period. The final state of wa-
Computer aspects of hydrological data analysis (including time data) in the context of water system (including transit times of water flow through segments of water system) is represented by dynamic network [FORD, FULKERSON 1969; ARONSON 1989]. From mathematical point of view it means that in pure or generalised network, real nonnegative function $T(k)$ defined on the set $A$ is introduced additionally. This function represents transit times of water flow through segments (arcs) of water system. In the construction of MDGNFM a technique of time-expanding of the underlying network based on the basic time unit was used [AHUJA et al. 1993]. Finally, in MDGNFM, the problem of determination of optimal dynamic water resources allocation (considering the transit time of water flow in the system) is formulated as an optimisation problem to determine minimum cost static flow in time-expanded network. The basic time unit based on which the network is expanding is in MDGNFM the greatest common divisor for all transit times and times of stages expressed in a common volume unit. In order to make it easier, the water flow velocity in the system is assumed to be constant within each day.

Example 1 (water allocation vs. water availability in a system)

Let us consider a water management system without water losses with the following network structure presented in Fig. 1.

The three dots symbolise the fact that between the user represented by arc $k_4$ and the user represented by $k_{13}$ there is a certain number of other users. The user represented by arc $k_{13}$ is situated the farthest. Let's assume: $u(k_{13}) = 12$, $u(k) = 1$ for environmental flow arcs, $u(k) \geq 13$ for all other arcs and $u(k) = 0$ for all arcs. Let's also assume that the time in each arc is identical, the time of water flow from node $s$ to node $v_{r+1}$ equals 2 days and that user $k_{13}$ is given the highest priority after environmental flows. Therefore, after satisfying the needs of environmental flows, water should be supplied to user $k_{13}$ in the first instance. Let's consider the hydrograph for the period of 7 days presented in Fig. 2.

Let's suppose that the problem of multiperiod optimisation with the time step of $T_p = 7$ days is being solved. The average daily value of water inflow into the system in the period of 7 days given in the hydrograph equals $Q_{av} = 13$. When solving the optimisation problem to determine a minimum cost static flow from the source $s$ to the sink $i$ in the network presented in Fig. 1, with the value of $V = Q_{av}$ and costs for arcs reflecting the hierarchy of water task priorities chosen, the following water allocations will be ob-
Let's assume that the time of each arc equals 1 hour and that the highest priority in the hierarchy of water tasks after environmental flow represented by arc $k_3$ is given to user $k_3$.

Let's assume that multiperiod optimisation problem is being solved. Let's assume that time step equals $T_p = 7$ days. The average daily value of water inflow into the system in each of the ten periods lasting the basic time step equals $Q_w = 13$. Let's solve the optimisation problem to determine a minimum cost static flow from the source $s$ to the sink $r$ in the network presented in Fig. 2 with the value of $V = Q_w$ and costs for arcs reflecting the hierarchy of chosen water task priorities. In each period the same result, i.e. allocating 1 unit to arc $k_3$ and 12 units to arc $k_4$, is obtained. Therefore, the demand of user $k_4$ is fully satisfied in each of the ten periods. Let's now assume the length of time step $T_p = 10$ days. When calculating the average daily value of water inflow into the system in each of the seven periods lasting the basic time step, the following sequence of numbers is obtained: 11.3, 15.3, 11.2, 13.1, 13.7, 11.1, 15.3. Let's solve the optimisation problem to determine a minimum cost static flow from the source $s$ to the sink $r$ in the network presented in Fig. 2, for each of the seven periods with the value of $V = Q_w$ and costs for arcs reflecting the hierarchy of chosen water task priorities. Environmental flow will be covered in each period and the allocations for user $k_4$ will equal respectively: 10.3, 12, 10.2, 12, 10.1, 12. Therefore, when the time step of 10 days is chosen, user $k_4$ will be satisfied in four out of seven periods and will not be satisfied in the other three periods. Different, contradictory information about user $k_4$ has been obtained depending on the chosen time step length. Consequently, in the static approach the final result may depend on the way historic data is grouped.

Such a problem is not present in the dynamic approach. When applying MDGNFM model, regardless of the fact whether the optimisation problem is solved for periods lasting 7 days or 10 days, the same final information about user $k_4$ is obtained. That is, the demand of user $k_4$ is covered only on two days, then for the next five days it is not covered and the situation repeats in cycles. All in all, in multiperiod dy-
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Dynamic approach the final result does not depend on the length of the chosen time step.

Example 3 (paths with varying accumulated times vs. water allocation)

This example is a modification of the example presented in [WOJAS 2008]. Let's consider a water management system with the following network structure presented in Fig. 5.

![Fig. 5. Fixed network structure of water system – example 3; source: own elaboration](image)

Let's assume: $u(k_3) = u(k_5) = 4, u(k_4) = 14, u(k_6) = 10, u(k_i) \geq 18$ for the other $i$ and $l(k) = 0$ for all arcs $k$. Arcs $k_3, k_5$ represent environmental flows with the value of 4 units per hour, arcs $k_4, k_6$ represent the users with a demand of 14 and 10 units per hour, respectively. Let's assume that water flow times expressed in hours equal: $T(k_4) = 3, T(k) = 1$ for all other arcs $k$. Hence, there are paths directed from $s$ to $v_3$ with different accumulated times. The time of the path consisting of arcs $k_1, k_3, k_5$ equals 5 hours, while the time of the path consisting of $k_1, k_2, k_5$ equals 3 hours. Let's also assume that the loss rate for arc $k_8$ equals 0.2 (20% of water is lost in this arc), while in the other arcs no water is lost. Let's choose the following hierarchy of water tasks (the order in which users are satisfied): $k_3 = k_5 > k_4 > k_i$ for the other $i$.

Let's consider the hydrograph for the period of 2 days presented in Fig. 6.

![Fig. 6. Hydrograph – example 3; source: own elaboration](image)

Let's use the dynamic approach. Tables 1 and 2 present optimal dynamic water allocations for the period of 2 days, which may be obtained by applying MDGNFM for the network presented in Fig. 5, with costs reflecting the hierarchy of chosen water task priorities and with the inflow values on individual days presented in Fig. 6.

<table>
<thead>
<tr>
<th>Arc No.</th>
<th>Time periods, h</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(0, 1)</td>
</tr>
<tr>
<td>$k_1$</td>
<td>18</td>
</tr>
<tr>
<td>$k_2$</td>
<td>0</td>
</tr>
<tr>
<td>$k_3$</td>
<td>4</td>
</tr>
<tr>
<td>$k_4$</td>
<td>0</td>
</tr>
<tr>
<td>$k_5$</td>
<td>4</td>
</tr>
<tr>
<td>$k_6$</td>
<td>0</td>
</tr>
<tr>
<td>$k_7$</td>
<td>4</td>
</tr>
<tr>
<td>$k_8$</td>
<td>0</td>
</tr>
<tr>
<td>$k_9$</td>
<td>4</td>
</tr>
</tbody>
</table>

Source: own elaboration.

The flow on the first day stabilises at the level of environmental flow. Water allocations for users $k_3, k_8$ equal 0 on the first day. On the second day water flow in the system stabilises after 12 hours reaching the stable level with water allocations for users $k_3, k_8$ covering their demands. As it may be seen in the tables, environmental flows are satisfied first (as soon as on the first day) by sending 4 uph (units per hour) from $s$ to $t$ using the fastest possible path (consisting of arcs $k_1, k_3, k_5, k_7, k_9$). The next, that is, second user $k_8$ is satisfied. On the second day, when the volume of the inflow into the system increases to 18 uph, 10 uph are sent to this user (which satisfies his demand) using the fastest possible path consisting of arcs $k_1, k_3, k_5$. It should be noticed that sending those units using the path $k_1, k_4, k_5$ and so through user $k_4$ would result in user’s $k_8$ suffering from the deficit (as the path $k_1, k_4, k_5$ is slower) and consequently, would disturb the hi-
erarchy of user priorities. Therefore, user $k_4$ is initially allocated with 4 uph and then (in the next hours) the number of allocated units increases. If the static approach were used in relation to the second day, the following allocations would be obtained: 4 uph for $k_3$, $k_7$, and $k_9$; 14 and 10 uph for $k_4$ and $k_8$, respectively. The same allocations as the ones obtained using the dynamic approach not sooner than after 8 hours. However, the static approach would not provide the information on how to reach such allocations and to not disturb the hierarchy of priorities. Allocating 14 uph to user $k_8$ as soon as at the beginning of the second day would result in user $k_4$, suffering from the deficit and disturb the hierarchy of priorities. The following question may be now asked: will optimal dynamic allocations after flow stabilisation and optimal static allocations be always the same? Generally, the answer is negative. Let's assume that in the example above the demand of user $k_3$ equalled 14 uph (not 10 uph). Using the dynamic approach we obtain the allocation of 0 for user $k_4$ on the second day, whereas the static approach gives us the allocation of 14 uph to user $k_4$ on the second day. So, the obtained results would be different. All in all, the static approach may lead to completely different final results from the ones obtained using the dynamic approach as it does not account for time differences in the paths along which water is sent to individual users.

**SUMMARY AND CONCLUSIONS**

This paper compares two mathematical approaches to the problem of determining optimal allocations of water resources in water management systems. Network static approach in a pure or generalised network has been compared to dynamic approach based on MDGNFM model presented in [WOJAS 2008]. The comparative analysis of both approaches has been performed using three working examples of water management systems. The following aspects have been discussed:

1) the ability of the model to guarantee the availability of water allocated to the user in the analysed period of time;
2) the influence of the time step length on the final result;
3) the ability of the model to account for paths with different accumulated times when distributing water.

In Example 1 attention has been paid to the fact that the static approach does not take into account the time it takes water to reach individual users of the water system and water allocation is based on the average value of water inflow into the system in one time step. The static model does not account for changes in the volume of water inflow in a time step as it uses the average values. Water allocation in the static approach is constant within a time step and water is allocated to the user as already available, not after the time it takes to reach the user as it is in fact. These features of the static approach may lead to the situation when the user will take over not 'his water' as the water allocated to him (according to the allocation in the static approach) will not in fact be available. It is particularly significant in the case of systems with high flow instability – when the user is allocated individual water rations and the flow time in the system is relatively long. Such problems do not occur in the dynamic approach based on MDGNFM model. The model accounts for both the time it takes water to reach individual users and the changes in water inflow into the system. According to the dynamic approach water is not allocated to users before it reaches him.

Example 2 shows that in multi-period static approach the final results may differ depending on the basic time step length. This results from averaging water inflow volume in the system in a given basic time step. The differences may be significant when the instability of water inflow into the system is high in the analysed period of time. Then, the problem arises of choosing the time step, at which the obtained result would be considered correct. Such a problem does not occur in the dynamic approach based on MDGNFM model. In this approach the final result does not depend on the time step length.

Example 3 illustrates the fact that the static approach may provide a different final result (different water allocation) than the dynamic approach because it does not take into account time differences between paths water is sent to individual users. In the static model all paths are treated as paths with identical accumulated times (to be precise, each time equals 0). Obviously, in the actual water system there are time differences between different paths (different ways of water transport). The network model of the system according to the dynamic approach (dynamic network) allows mapping mathematically the actual structure of the times it takes water to flow through the system segments. When distributing water according to the dynamic approach, first to occur is the period of flow dynamic stabilisation – as demonstrated in Example 3. That period is not taken into account in the static approach at all. According to the static approach water allocations are constant in the analysed period of time. In the case of frequent changes in water inflow to the system, the stabilisation periods constitute a significant part of the analysed period of time. As a result, the water allocations obtained in both approaches may significantly differ from each other.

When comparing both approaches, the following general conclusion may be formulated: when the volume of water inflow to the system is constant and the times of all arcs are the same (there are no time differences between different paths to individual users), both approaches, i.e. the static approach and the dy-
Porównanie statycznego i dynamicznego podejścia sieciowego w modelach alokacji zasobów wodnych dla rzek o dużej zmienności przepływów

STRESZCZENIE

Słowa kluczowe: program alokacja zasobów wodnych, bilans wodno-gospodarczy, optymalizacja sieciowa, przepływy w sieciach, sieci dynamiczne

W pracy dokonano porównania dwóch matematycznych podejść sieciowych do problemu wyznaczania optymalnych alokacji zasobów wodnych w systemach wodno-gospodarczych. Porównano standardowe podejście statyczne, bazujące na modelu statycznego przepływu w sieci czystej lub uogólnionej, z podejściem dynamicznym, bazującym na modelu MDGNFM zaprezentowanym w pracy WOJAS [2008]. Porównania dokonano na trzech roboczych przykładach systemów wodno-gospodarczych. Omówione zostały następujące aspekty: możliwość zagwarantowania w modelu dostępności przydzielonej użytkownikowi wody w analizowanym okresie; wpływ wyboru długości kroku czasowego na wynik końcowy; możliwość uwzględniania siećek o różnych czasach sumarycznych przy rozdziale wody. W wyniku analizy porównawczej za bardziej odpowiednie w przypadku systemów wodno-gospodarczych o dużej zmienności i dynamice przepływów uznanano podejście dynamiczne oparte na modelu MDGNFM.